Optimal sensor and actuator selection in dynamic networks

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Motivation

- Select **optimal subset** of potential sensors/actuators
  - Sensor/Actuator types
  - Sensor/Actuator locations
- Applications
  - Heterogeneous robotic networks
  - Phasor Measurement Units in power networks
  - Sensors and actuators in flexible aircraft wings

Actuator Selection

Model

- Linear system with many actuators

$$\dot{x} \;=\; A\,x \,+\, B_1\,d \,+\, B_2\,u$$

Performance measure

- Steady-state variance amplification

$$\lim_{t \to \infty} \mathcal{E}\left(x^T(t)\,Q\,x(t) \,+\, u^T(t)\,R\,u(t)\right)$$

Objective

- Identify **row-sparse state-feedback controller**

$$u \;=\; -Kx$$

to balance:

Performance: **variance amplification**

Sparsity: **number of actuators**

Optimization problem

$$\text{minimize} \quad J(K) \quad + \quad \gamma \sum_{i=1}^{m} \|\mathrm{e}_i^T K\|_2$$

$J(K)$ ↓ **variance amplification**; $\gamma \sum_{i=1}^{m} \|\mathrm{e}_i^T K\|_2$ ↓ **sparsity-promoting penalty function**

$\gamma > 0$ – variance amplification vs sparsity tradeoff

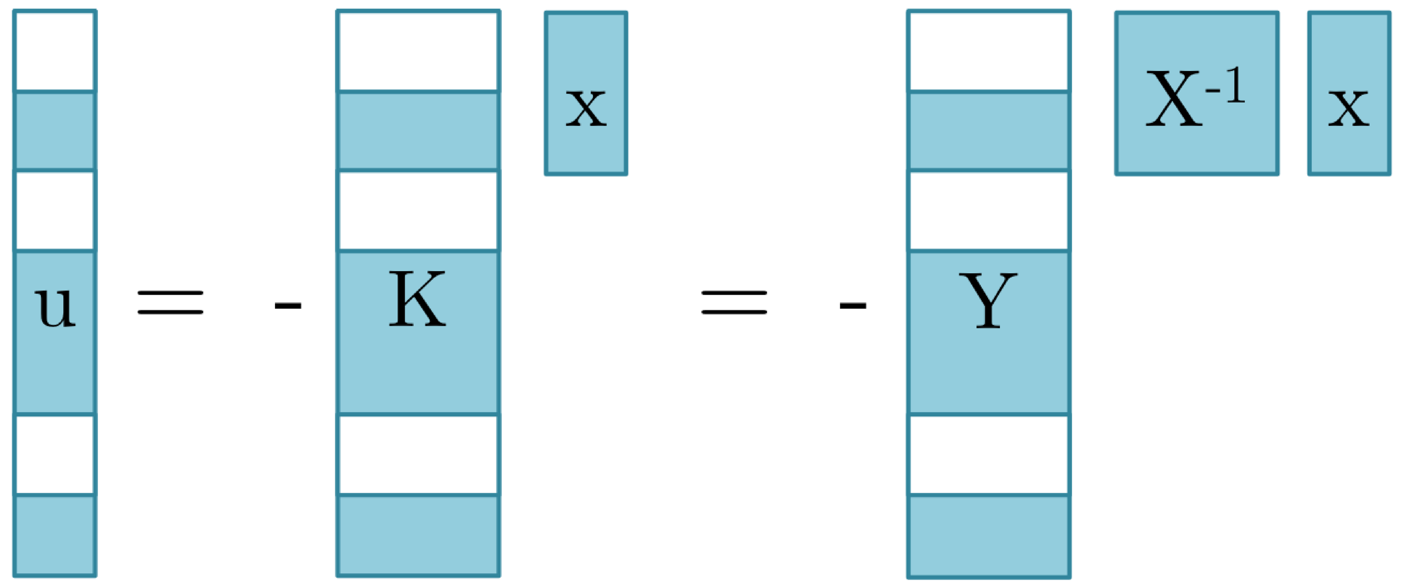
Convex Formulation

Challenge

- $J(K)$ – non-convex function of K

Key observation

- Change of variables $Y := KX$
  - Yields convex dependence of $J(K)$ on X and Y
  - Preserves row-sparse structure



Semidefinite Program

$$\begin{array}{ll} \text{minimize} & J(X,Y) \,+\, \gamma \displaystyle\sum_{i=1}^{m} \|\mathrm{e}_i^T Y\|_2 \\ \text{subject to} & AX + XA^T - B_2 Y - Y^T B_2^T + B_1 B_1^T = 0 \\ & X \succ 0 \end{array}$$

Efficient Algorithm

Alternating Direction Method of Multipliers

Form augmented Lagrangian

$$\mathcal{L}_\rho(X,Y,\Lambda) \;:=\; J(X,Y) \,+\, \gamma\, g(Y) \,+\, \phi_\rho(X,Y,\Lambda)$$

by dualizing and penalizing linear constraint, $h(X,Y)$,

$$\phi_\rho(X,Y,\Lambda) \;:=\; \text{trace}\left(\Lambda^T h(X,Y)\right) \,+\, \frac{\rho}{2}\,\|h(X,Y)\|_F^2$$

Iteratively solve tractable subproblems

$$\begin{array}{lll} X_{k+1} &=& \underset{X}{\arg\min}\;\; \mathcal{L}_\rho(X,\, Y_k,\, \Lambda_k) \qquad \textit{Projected Descent} \\ Y_{k+1} &=& \underset{Y}{\arg\min}\;\; \mathcal{L}_\rho(X_{k+1},\, Y,\, \Lambda_k) \qquad \textit{group LASSO} \\ \Lambda_{k+1} &=& \Lambda_k \,+\, \rho\, h(X_{k+1},\, Y_{k+1}) \end{array}$$

Sensor Selection

Estimate state x from noisy output y

$$\begin{array}{rcl} \dot{x} &=& A\,x \,+\, B_1\,d \\ y &=& C\,x \,+\, \eta \end{array}$$

- Identify **observer gain** to balance

Performance: **variance amplification**

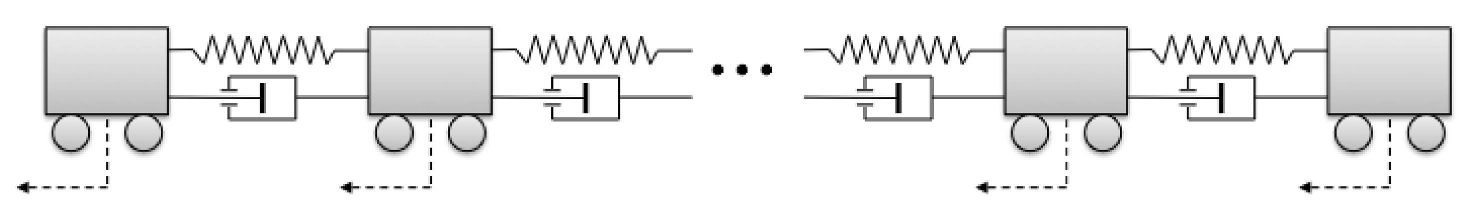
Sparsity: **number of sensors**

Key point

- Can be brought to actuator selection problem

A Sensor Selection Example

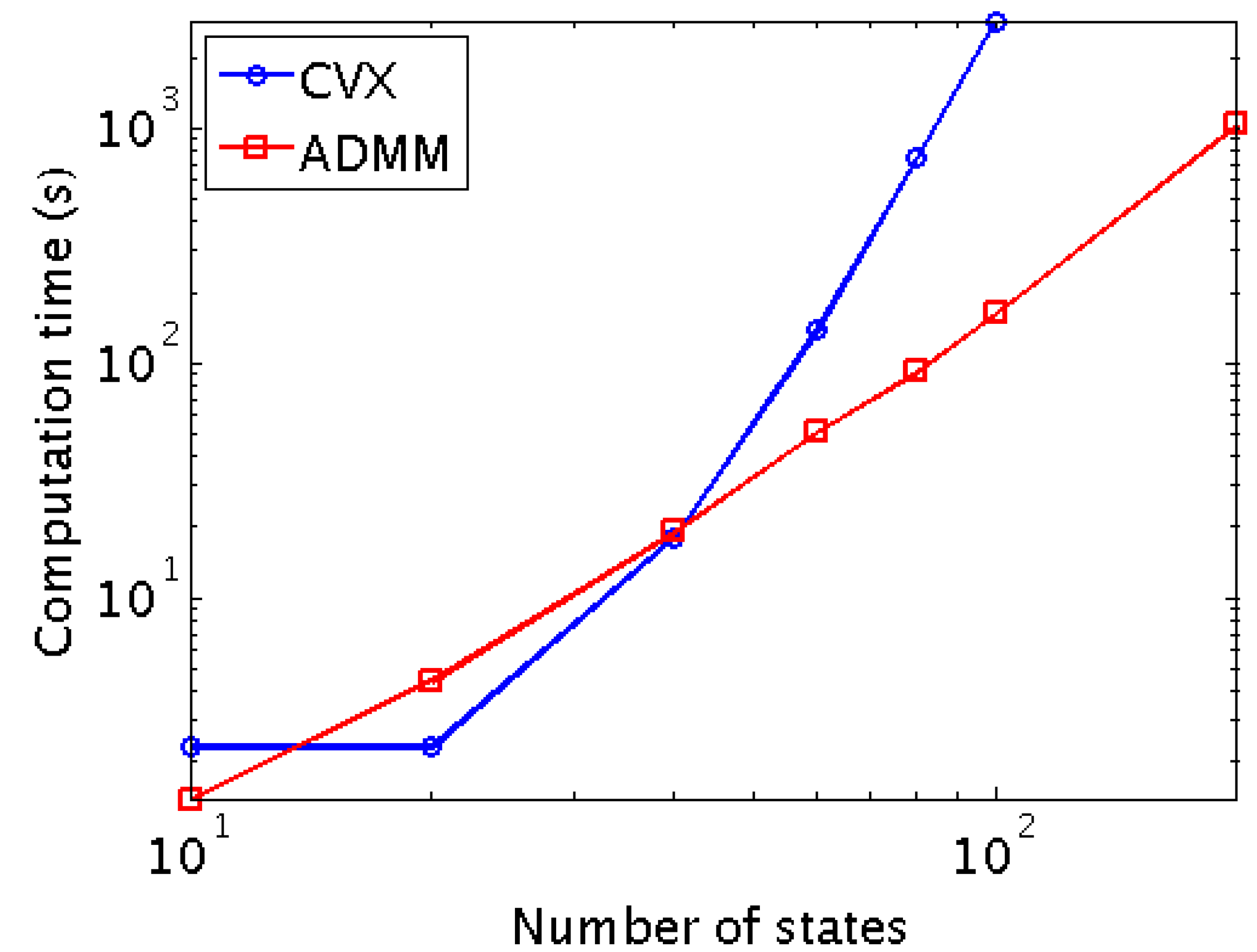
Vehicular formation



Objective

- Optimal GPS placement

CVX vs ADMM for $\gamma = 100$:



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