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Controller architectures: Tradeoffs between performance and structure ☆

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A B S T R A C T

This review article describes the design of static controllers that achieve an optimal tradeoff between closed-loop performance and controller structure. Our methodology consists of two steps. First, we identify controller structure by incorporating regularization functions into the optimal control problem and, second, we optimize the controller over the identified structure. For large-scale networks of dynamical systems, the desired structural property is captured by limited information exchange between physical and controller layers and the regularization term penalizes the number of communication links. Although structured optimal control problems are, in general, nonconvex, we identify classes of convex problems that arise in the design of symmetric systems, undirected consensus and synchronization networks, optimal selection of sensors and actuators, and decentralized control of positive systems. Examples of consensus networks, drug therapy design, sensor selection in flexible wing aircrafts, and optimal wide-area control of power systems are provided to demonstrate the effectiveness of the framework.

1. Introduction

Large-scale networks of dynamical systems are ubiquitous in modern applications. Systems of this type arise in applications ranging from distributed power generation, to deployment of teams of robotic agents, to control of segmented mirrors in extremely large telescopes, to control of fluid flows around wind turbines and vehicles. One of the major challenges in the study of networks of dynamical systems is the development of analytical and computational methods for their tractable analysis and design.

The optimal control of linear systems with quadratic performance measures, such as LQR, $\mathcal{H}_2$, and $\mathcal{H}_\infty$, is a cornerstone of systems theory. This framework provides a systematic way to balance closed-loop performance, robustness, and control effort. In the conventional formulation, an optimal controller is designed to minimize some measure of the amplification from the sources of excitation to a regulated output which penalizes both the state and the control effort. These optimal controllers are typically implemented in a centralized fashion, which is not feasible in many emerging applications.

Recent technological advances have allowed the individual components of large-scale systems to be equipped with their own sensing, actuation, communication, computation, and decision making capabilities. Advances in Micro-Electro-Mechanical-Systems (MEMS) have enabled the development of arrays of sensors and actuators that can interact with one another. Strings of vehicular platoons, unmanned aerial vehicles (UAVs), and robotic agents constitute another set of examples of large-scale autonomous systems [13,79]. In many of these applications, the scale of the problem, constraints on computing and communication resources, and the wide-spread of sensing and actuating capabilities pose additional requirements on controller complexity. Typically, these cannot be addressed using tools from standard optimal control theory. For example, a dense state-feedback controller resulting from the LQR framework would impose a prohibitive communication burden in large-scale networks. This is because forming every control input requires information from every subsystem in the distributed plant. The cost of creating and maintaining communication links makes such an all-to-all topology infeasible in most large-scale and distributed systems.

This has motivated the design of structured (both decentralized and distributed) controllers. Early efforts have centered on the design of decentralized strategies [97,101] and, during the last fifteen years, the emphasis has shifted to the design of distributed controllers

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[2,21,111,20,60–65,102,3,96,8,99,106,36,72,71,100,85,42,112,82]. Two major issues have emerged: the identification of convex classes of structured control problems and the optimal control design under *a priori* specified structural constraints.

Optimal control problems are often reformulated using Youla parameterization [44,34]. The mapping from the controller to the Youla parameter is nonlinear which typically compromises convexity of the structural constraints in the distributed setup. It is thus important to identify subspaces which remain invariant under this nonlinear mapping for distributed systems. In [111,3], the subspaces of *cone* and *funnel causal* systems have been introduced; these describe how information from every controller propagates through the distributed system. For spatially invariant systems, the design of quadratically optimal controllers can be cast into a convex problem if the information in the controller propagates at least as fast as in the plant [111,3]. A similar but more general algebraic characterization of the constraint set was introduced and convexity was established under the condition of quadratic invariance in [96]. Other classes of convex distributed control problems include partially nested systems [52,109,110], poset-causal systems [99,100], and positive systems [41,107,12,35,94,17,95].

Since most of these convex formulations are expressed in terms of the impulse response parameters, they do not lend themselves easily to state-space characterization. Apart from very special instances, the optimal distributed design problem remains challenging. For poset-causal systems, explicit Riccati-based solutions for the optimal decentralized state-feedback problem were obtained in [99,100]. For a two-player problem with block-triangular state-space matrices, the optimal decentralized output-feedback solution was recently provided in [64,65]. Characterizing the structural properties of optimal distributed controllers is another important challenge. For spatially invariant systems, the quadratic optimal controllers are also spatially invariant and the information from other subsystems is exponentially discounted with the distance between the controller and the subsystems [2]. For systems on graphs, this spatially decaying property was studied in [83,84] and it motivates the search for inherently localized controllers.

Recently, it has been demonstrated that the *design of controller architectures* can have a more profound impact on the closed-loop performance than the optimal design under a given pre-specified architecture [1]. In [39,68], tools and ideas from control theory, optimization, and compressive sensing have been combined to systematically address the challenge of designing controller architectures. The proposed approach introduces regularized versions of standard optimal control problems and aims to strike a balance between closed-loop performance and controller complexity. For example, when the state vector and control inputs can be partitioned into subvectors that correspond to separate subsystems, promoting sparsity of the feedback gain matrix limits information exchange between the physical system and the controller. Sparse controller architectures can be designed by augmenting standard quadratic performance measures with sparsity-promoting penalty functions which serve as measures of controller complexity. Such an approach has received much recent attention [39,68,98,37,73,74,76,75].

Alongside the sparse feedback synthesis, the critical question of sensor and actuator selection has been recently considered in [92,28]. Although, in general, finding the solution to this problem requires an intractable combinatorial search, by drawing upon recent developments in sparse representations this problem can be cast as a semidefinite program (SDP). Moreover, it is also of interest to study problems where it is desired to optimize a linear function of some design variable to which regularization or convex constraints are applied [117]. This broader framework covers a wide variety of problems ranging from wide-area and distributed PI control of power networks [32,33,114,115], to combination drug therapy for HIV treatment [24], to edge addition [48–50] and leader selection [91,43,69,16,23] in consensus networks.

Several recent efforts have focused on establishing convexity for classes of these problems and on developing efficient algorithms for optimal controller design for both convex and nonconvex problems. Convex structured optimal control problems include symmetric modifications to symmetric linear systems [39,25,29], diagonal modifications to positive systems [24,23], optimal sensor and actuator selection [92,28], and edge addition to undirected consensus [48–50] and synchronization [40] networks. Algorithmic developments have employed alternating direction method of multipliers [68,28], proximal gradient and Newton methods [50], as well as first- and second-order method of multipliers [26,27] to efficiently perform identification of controller structure and structured feedback synthesis.

Our presentation is structured as follows. In Section 2, we provide motivation and background and highlight challenges that arise in structured feedback synthesis. In Section 3, we formulate the problem of managing controller complexity via regularization and describe different sparsity-promoting regularizers. In Section 4, we introduce generalized problem formulation and summarize several classes of regularized optimal control problems which admit convex characterizations. These include optimal design of sparse symmetric systems, sparse synthesis of undirected consensus and synchronization networks, optimal selection of sensors and actuators, selection of influential nodes in the networks of single-integrators, and combination drug therapy design for HIV treatment. In Section 5, we provide examples to illustrate the framework and its utility. We conclude with remarks in Section 6.

2. Structured optimal control

The notion of controller structure can have different connotations. To motivate the study of structured optimal control, we begin with a discussion of networks of dynamical systems. Traditional design techniques, such as LQR, $\mathcal{H}_2$, and $\mathcal{H}_\infty$, require centralized implementation of the resulting controllers. For large-scale systems, the computational and communication costs associated with such a centralized implementation may be prohibitively high. It is thus of interest to design controllers with distributed structure and sparse communication topologies. We first draw a connection between sparsity of the feedback gain matrix and the induced communication topology, and then highlight challenges that arise in the design of structured state-feedback controllers.

2.1. Motivation and background

We study linear time-invariant systems

$$\dot{x} = A\,x + B_1\,d + B_2\,u \tag{1}$$

where x is the state, d is the disturbance, and u is the control. To motivate our developments, let us for a moment assume that (1) contains N individual subsystems, each with a local state and control inputs, and let B_2 be a block-diagonal matrix. By partitioning the state and control input vectors into subvectors corresponding to each subsystem, $x := [x_1^T \cdots x_N^T]^T$ and $u = [u_1^T \cdots u_N^T]^T$, we can write the subsystem dynamics as,

$$\dot{x}_i = A_{ii}\,x_i + \sum_{j \neq i} A_{ij}\,x_j + B_{1i}\,d + B_{2,ii}\,u_i. \tag{2a}$$

The block-sparsity pattern of A determines the interaction topology between subsystems; when A_{ij} is zero, subsystem j has no direct effect on the evolution of the state of subsystem i.

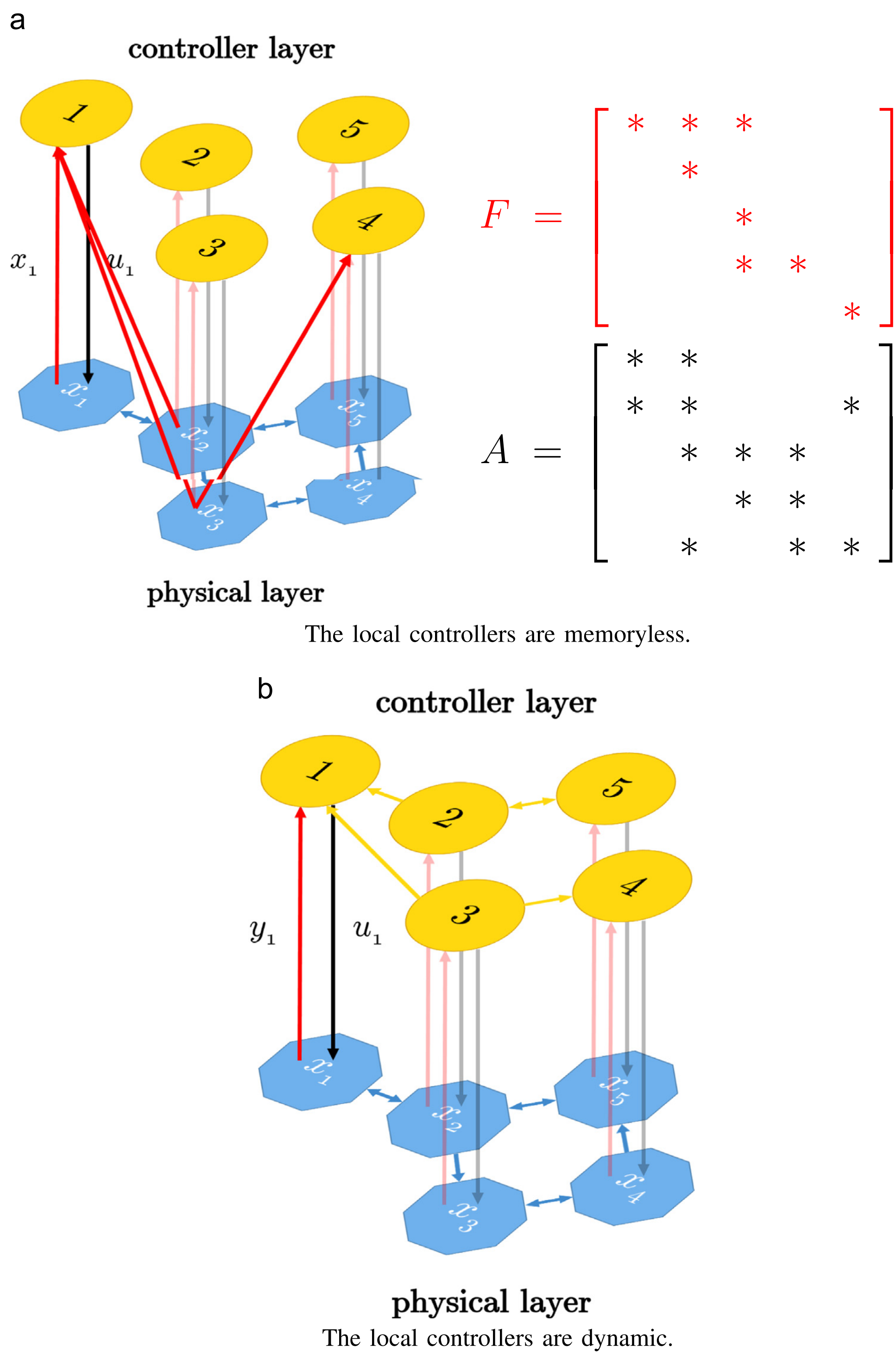

Fig. 1. A network of 5 dynamical systems with associated local controllers.

With each subsystem we associate a controller that specifies the control input u_i. Standard optimal control techniques typically induce a communication topology which requires every local controller to have access to the state of every subsystem. In large-scale networks of dynamical systems, this may impose significant communication burden and implementation may be prohibitively expensive. It is thus of interest to explore the design of feedback laws that utilize *limited* information exchange within a large-scale network.

Under linear state-feedback $u = -Fx$ the dynamics (2a) become,

$$\dot{x}_i = A_{ii}\, x_i + \sum_{j \neq i} A_{ij}\, x_j + B_{1i}\, d - B_{2,ii} \sum_j F_{ij}\, x_j. \tag{2b}$$

Thus, the block-sparsity pattern of the feedback gain matrix F determines the communication topology of the static controller: forming the control input u_i requires access to the states of each subsystem j for which F_{ij} is nonzero.

Fig. 2. Mass–spring system on a line.

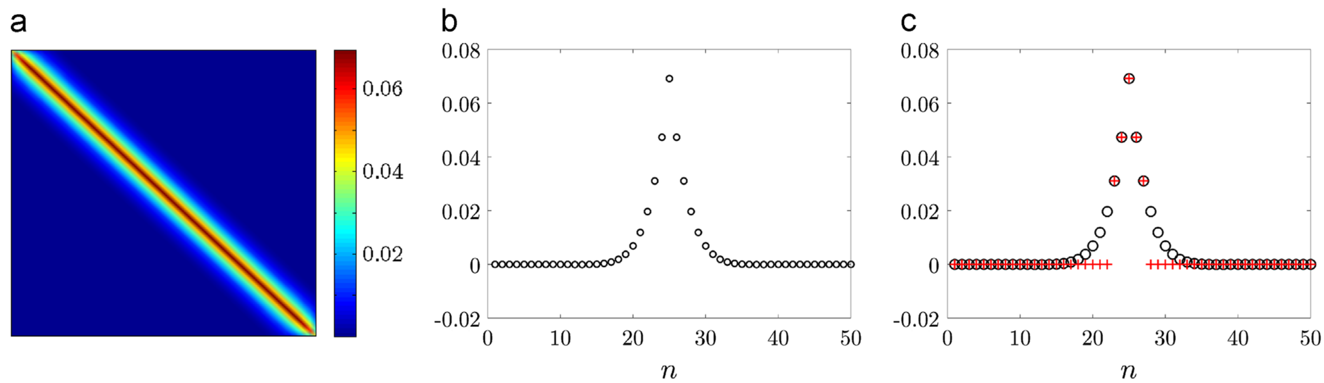

Fig. 3. (a) The optimal centralized position feedback gain matrix F_p in the system with 50 masses. Both F_p and F_v (not shown) have almost constant diagonals (modulo edges) and exponential off-diagonal decay. (b) Optimal centralized position gains for the middle mass $n = 25$. (c) Truncation of the optimal centralized position gains for the middle mass $n = 25$.

Fig. 1a illustrates a network of coupled subsystems, associated controller topology, and the sparsity patterns of the corresponding matrices A and F. The subsystems in the physical layer are represented by blue octagons; their interaction topology is marked by the blue arrows and captured by the sparsity pattern of the matrix A. Each local controller is represented by a yellow circle; the structure of the information exchange network between the two layers is marked by the red arrows and captured by the sparsity pattern of the feedback gain matrix F.

In the more general setup where the local controllers are dynamic (perhaps because they estimate the subsystem's state rather than directly measure it), it is important to determine the order of local controllers as well as the structure of the information exchange network in the controller layer; see Fig. 1b for an illustration. Recent advances have been made for particular classes of systems [64,65], but addressing these questions in general remains an open challenge.

2.1.1. An example

For the system with N masses shown in Fig. 2, the state vector is determined by $x = [p^T\ v^T]^T$, where p and v are the vectors of positions and velocities of all masses, respectively. Setting all masses and spring constants to unity and partitioning matrices in the state-space model (1) conformably with the partition of x yields

$$A = \begin{bmatrix} 0 & I \\ T & 0 \end{bmatrix}, \quad B_1 = B_2 = \begin{bmatrix} 0 \\ I \end{bmatrix},$$

where T is an $N \times N$ tridiagonal Toeplitz matrix with -2 on its main diagonal and 1 on its first sub- and super-diagonal. We set the state and control performance weights to $Q = I$ and $R = 10I$, respectively. In the absence of the structural constraints, the solution to the Riccati equation yields the centralized controller $F := [F_p \quad F_v]$ with dense position and velocity feedback gain matrices F_p and F_v,

$$\begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \\ u_4(t) \end{bmatrix} = -\underbrace{\begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix}}_{F_p} \begin{bmatrix} p_1(t) \\ p_2(t) \\ p_3(t) \\ p_4(t) \end{bmatrix} - \underbrace{\begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix}}_{F_v} \begin{bmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \\ v_4(t) \end{bmatrix}.$$

Even though these matrices are populated with non-zero elements, the gains that are used to form control actions for individual masses display interesting patterns. Fig. 3 illustrates the optimal centralized position feedback gain matrix F_p in the system with 50 masses. Apart from the edges, both F_p and F_v (not shown) have almost constant diagonals and exponential off-diagonal decay.

For spatially invariant systems, the optimal controllers with respect to quadratic performance indices (e.g., LQR, $\mathcal{H}_2$, $\mathcal{H}_\infty$) are also spatially invariant and they exponentially discount information with spatial distance [2]. Moreover, it has been suggested that optimal controllers for spatially decaying systems over general graphs also possess spatially decaying property [83,84]. This motivates the search for inherently *localized* controllers and suggests that localized information exchange in the distributed controller may provide a viable strategy for controlling large-scale systems. For example, one could search for optimal controllers that are subject to the condition that they communicate only to a subset of other subsystems. However, incorporating structural restrictions on F significantly complicates the design problem and it is difficult to provide error bounds on the deviation from optimality if one were to truncate the information dependence of every controller (e.g., by confining information exchange within a pre-specified radius or by removing gains whose magnitude does not exceed a certain threshold; see Fig. 3c for an illustration). Furthermore, it has been recognized that the truncation of the centralized controller could significantly compromise the closed-loop performance and even yield a controller that does not guarantee closed-loop stability [83,68].

2.2. Structured optimal control

In what follows, we use the $\mathcal{H}_2$ norm to quantify the closed-loop performance. In the centralized case, the optimal control law for LTI systems is given by static state-feedback. Even though it is not clear if the optimal *distributed* controller is also memoryless, we use a class of static controllers to highlight some of the challenges that arise in structured control synthesis.

The closed-loop dynamics resulting from system (1) with state-feedback controller $u = -Fx$ and the performance output z are given by

$$\begin{aligned} \dot{x} &= (A - B_2 F)\, x + B_1\, d \\ z &= \begin{bmatrix} Q^{1/2} \\ -R^{1/2} F \end{bmatrix} x \end{aligned} \qquad \text{(CL)}$$

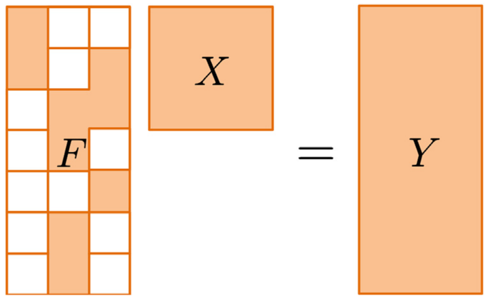

Fig. 4. The change of variables that casts the unstructured state-feedback problem as an SDP, in general, does not preserve the structural properties of F.

where $C := \begin{bmatrix} Q^{1/2} & 0 \end{bmatrix}^T$ and $D := \begin{bmatrix} 0 & R^{1/2} \end{bmatrix}^T$, with standard assumptions on stabilizability and detectability of the pairs (A, B_2) and $(A, Q^{1/2})$. The feedback gain matrix $F \in \mathbb{R}^{m \times n}$ is the design variable, $Q = Q^T \succcurlyeq 0$ and $R = R^T \succ 0$ are the state and control performance weights, and the performance metric is the steady-state amplification from the white stochastic disturbance d to the performance output z,

$$J(F) := \lim_{t \to \infty} \mathbf{E}\left(z^T(t)\, z(t)\right) = \lim_{t \to \infty} \mathbf{E}\left(x^T(t) Q\, x(t) + u^T(t) R\, u(t)\right).$$

This quantity is determined by the square of the $\mathcal{H}_2$ norm and it can be expressed as a function of the feedback gain F as

$$J(F) = \begin{cases} \text{trace}\Big((Q + F^T R\, F)\, X\Big), & F \text{ stabilizing} \\ +\infty, & \text{otherwise} \end{cases}$$

where X is the closed-loop controll ability gramian,

$$(A - B_2 F)\, X + X\, (A - B_2 F)^T + B_1 B_1^T = 0. \tag{3}$$

In the absence of the structural constraints on the matrix F, the optimal $\mathcal{H}_2$ feedback gain is determined by the linear quadratic regulator and it can be computed via the solution of the algebraic Riccati equation. However, as we describe next, incorporating structural restrictions on F significantly complicates the design problem.

The design of the optimal state-feedback gain F, subject to constraints on its sparsity pattern (equivalently, on the communication topology in Fig. 1a), has a rich history and was recently revisited in [38,66]. Let the subspace $\mathcal{S}$ encapsulate these structural constraints and let us assume that there is a stabilizing $F \in \mathcal{S}$. The optimal control problem of determining stabilizing $F \in \mathcal{S}$ that minimizes the $\mathcal{H}_2$ norm of the closed-loop system (CL) can be formulated as

$$\begin{array}{ll} \text{minimize} & J(F) \\ \text{subject to} & F \in \mathcal{S} \end{array} \tag{SH2a}$$

and brought into the following form:

$$\begin{array}{ll} \underset{X, F}{\text{minimize}} & \text{trace}\Big((Q + F^T R\, F)\, X\Big) \\ \text{subject to} & (A - B_2 F)\, X + X\, (A - B_2 F)^T + B_1 B_1^T = 0 \\ & X \succ 0, \;\; F \in \mathcal{S}. \end{array} \tag{SH2b}$$

In the absence of the structural constraint $F \in \mathcal{S}$, a standard change of variables [34]

$$Y := F\, X \tag{4}$$

can be used to express the square of the $\mathcal{H}_2$ norm as,

$$J(X, Y) = \text{trace}\,(Q\, X) + \text{trace}\,(R\, Y X^{-1}\, Y^T)$$

and the Schur complement can be employed to cast the optimal state-feedback $\mathcal{H}_2$ control problem as an SDP,

$$\begin{array}{ll} \underset{X, Y, Z}{\text{minimize}} & \text{trace}(Q\, X) + \text{trace}(R\, Z) \\ \text{subject to} & (A\, X - B_2\, Y) + (A\, X - B_2\, Y)^T + B_1 B_1^T = 0 \\ & \begin{bmatrix} Z & Y \\ Y^T & X \end{bmatrix} \succcurlyeq 0. \end{array}$$

Since X is positive definite, it is invertible, and the optimal centralized (i.e., unstructured) F is determined by $F_c = Y X^{-1}$. This centralized solution coincides with the linear quadratic regulator, which can be explicitly determined by $F_c = R^{-1} B_2^T P$ where P is the unique positive definite solution of the algebraic Riccati equation, $A^T P + PA + Q - PB_2\, R^{-1} B_2^T P = 0$.

The above change of variables is, in general, not suitable for imposing structure on F. Although the constraint on the feedback gain matrix $F \in \mathcal{S}$ is linear and thus convex, the corresponding constraint on X and Y is bilinear, $Y X^{-1} \in \mathcal{S}$. This makes it difficult to translate the sparsity patterns of F to the sparsity patterns of X and Y (see Fig. 4 for an illustration), thereby limiting the use of these coordinates for structured design problems. By restricting X to be diagonal, the sparsity structure of F coincides with the sparsity structure of Y. However, this may introduce considerable conservatism in the design and may not even lead to a feasible SDP characterization (even when the original nonconvex problem is feasible).

3. Design of sparse controller architecture via regularization

The communication architecture $\mathcal{S}$ of the state-feedback controller in (SH2b) is fixed and *a priori* specified which may impose limits on the achievable performance. For problems where the communication topology is *not* fixed, it is desirable to *design* a favorable communication topology while promoting sparsity of the communication links. To achieve this, an optimization framework which augments the $\mathcal{H}_2$ objective function with a penalty on the sparsity of the feedback gain matrix (i.e., the number of communication links) was introduced in [39,68].

3.1. Structure identification

Our objective is to design controller architecture that achieves a desired tradeoff between the quadratic performance of closed-loop system (CL) and the sparsity of the feedback gain F. To address this challenge we consider a regularized optimal control problem

$$\underset{F}{\text{minimize}} \quad \underbrace{J(F)}_{\substack{\textbf{closed-loop} \\ \textbf{performance}}} \; + \; \gamma\, \underbrace{g(F)}_{\substack{\textbf{controller} \\ \textbf{structure}}} \tag{SP}$$

where $J(F)$ is the $\mathcal{H}_2$ norm of (CL). In contrast to (SH2b), no structural constraints are imposed on F in (SP); instead, the objective is to manage structure of the controller by introducing a regularization term $g(F)$ into the optimal control problem. The non-negative regularization parameter γ encodes the emphasis on controller structure relative to the closed-loop performance. For $\gamma = 0$, the centralized LQR solution is obtained. As γ increases, larger emphasis is placed on obtaining the feedback gain F that satisfies some additional structural requirements; see Fig. 5 for an illustration.

Problem (SP) is difficult to solve directly because J is typically a nonconvex function of F and g is convex but not differentiable.

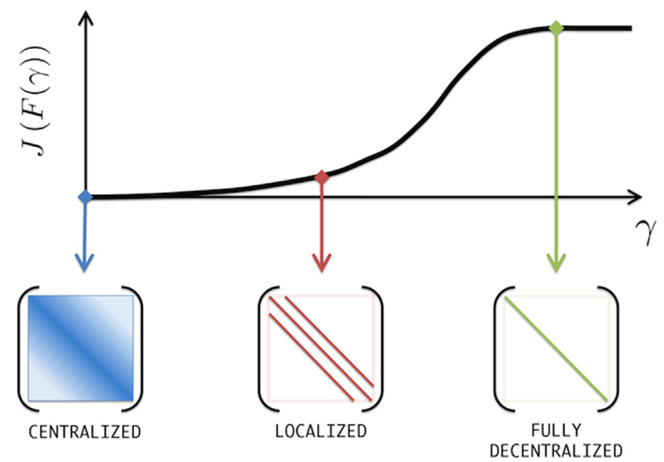

Fig. 5. Increased emphasis on sparsity encourages sparser control architectures at the expense of deteriorating the closed-loop performance. For $\gamma = 0$ the optimal centralized controller F_c is obtained from the positive definite solution of the algebraic Riccati equation. Control architectures for $\gamma > 0$ are determined by $F(\gamma) := \text{argmin}_F (J(F) + \gamma\, g(F))$ and they depend on interconnections in the distributed plant and the state and control performance weights Q and R.

While the nonlinear change of coordinates (4) yields a convex dependence of J on X and Y, in general, it introduces a nonconvex dependence of the regularization term g on these optimization variables.

Introduction of an auxiliary variable to the regularized optimal control problem (SP),

$$\begin{array}{ll} \underset{F,G}{\text{minimize}} & J(F) + \gamma\, g(G) \\ \text{subject to} & F - G = 0 \end{array}$$

facilitates the use of splitting methods that exploit the respective structures of J and g in (SP). In [68], the alternating direction method of multipliers [9] was used to find a solution by iteratively solving simpler subproblems over F and G. The F-minimization problem is typically nonconvex but smooth; in contrast, the G-minimization problem is non-differentiable but convex and it often admits an explicit solution. More recently, the proximal methods were combined with the method of multipliers to systematically address the challenge of step-size selection in the augmented Lagrangian and provide convergence guarantees [26].

3.1.1. *Sparsity-promoting regularizers*

The elementwise sparsity of the feedback gain matrix F can be promoted by incorporating the cardinality function into the optimal control problem (SP),

$$g_0(F) = \mathbf{card}(F). \tag{5a}$$

This regularizer counts the number of nonzero elements in F and it yields a combinatorial optimization problem (SP) whose solution typically requires an intractable combinatorial search. A weighted ℓ_1 penalty on the individual elements $F_{ij} \in \mathbb{R}$ of the feedback gain matrix F,

$$g_1(F) = \|W \circ F\|_1 = \sum_{i,j} w_{ij} |F_{ij}| \tag{5b}$$

provides a convex proxy for promoting elementwise sparsity of the matrix F [14]. Here, W is the matrix whose elements are determined by the non-negative weights w_{ij} and $\circ$ is the elementwise matrix multiplication. The weights w_{ij} can be selected to place larger relative penalties on certain elements of F. Furthermore, if some information comes for free, the corresponding w_{ij} can be set to zero. Similarly, the sum of the Frobenius norms of the submatrices $F_{ij} \in \mathbb{R}^{m_i \times n_j}$,

$$g_2(F) = \sum_{i,j} w_{ij} \|F_{ij}\|_F \tag{5c}$$

enhances sparsity at the level of submatrices [118]. Here, the feedback gain F can be partitioned into submatrices F_{ij} that need not be of the same dimension, and the weights $w_{ij} \geq 0$ specify the emphasis on sparsity of individual blocks. In particular, when it is desired to promote the row-sparsity of F, g_2 simplifies to

$$g_3(F) = \sum_i w_i \|e_i^T F\|_2 \tag{5d}$$

where e_i is the standard ith basis vector in $\mathbb{R}^m$ and $\|\cdot\|_2$ is the Euclidean norm. This penalty promotes limited use of input control channels and can thus be used as a convex proxy for optimal selection of actuators [92,28].

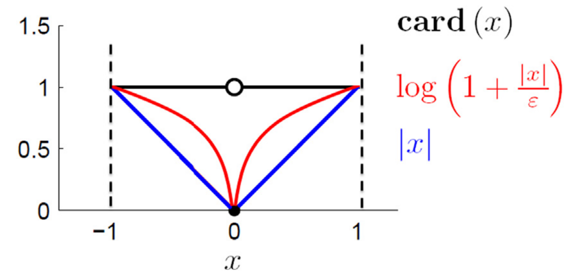

Fig. 6. Cardinality function of a scalar variable x and the corresponding absolute value and logarithmic approximations on $x \in [-1, 1]$.

The ℓ_1 norm is the largest convex function that underestimates the cardinality function on the box with edges of unit length [10]; see Fig. 6 for an illustration in the scalar case. Both the ℓ_1 norm and its weighted version are convex relaxations of $\mathbf{card}(F)$. On the other hand, better approximation can be obtained with nonconvex functions, e.g., the sum-of-logs,

$$g_4(F) = \sum_{i,j} \log\left(1 + \frac{|F_{ij}|}{\varepsilon}\right), \quad 0 < \varepsilon \ll 1. \tag{5e}$$

The weighted ℓ_1 norm attempts to bridge the difference between the ℓ_1 norm and the cardinality function. In contrast to the cardinality function that assigns the same cost to any nonzero element, the ℓ_1 norm penalizes more heavily the elements of larger magnitudes. The positive weights can be chosen to counteract this magnitude dependence of the ℓ_1 norm. For example, if the weights w_{ij} are inversely proportional to the magnitude of F_{ij},

$$\begin{cases} w_{ij} = 1/|F_{ij}|, & F_{ij} \neq 0 \\ w_{ij} = \infty, & F_{ij} = 0 \end{cases}$$

then there is no difference between the weighted ℓ_1 norm of F and the cardinality function of F. This scheme, however, cannot be implemented, because the weights depend on the unknown feedback gain. A re-weighted algorithm that solves a sequence of weighted ℓ_1 optimization problems was proposed in [14]. In this, sequential linearization of the sum-of-logs function is used and the weights are determined by the solution of the optimization problem in the previous iteration. This algorithm has provided an effective heuristics for promoting sparsity in many emerging applications.

Additional intuition about the role of sparsity-promoting regularizers can be gained by considering a problem in which it is desired to find the sparsest feedback gain that provides a given level of $\mathcal{H}_2$ performance $\sigma > 0$,

$$\begin{array}{ll} \text{minimize} & \mathbf{card}(F) \\ \text{subject to} & J(F) \leq \sigma. \end{array}$$

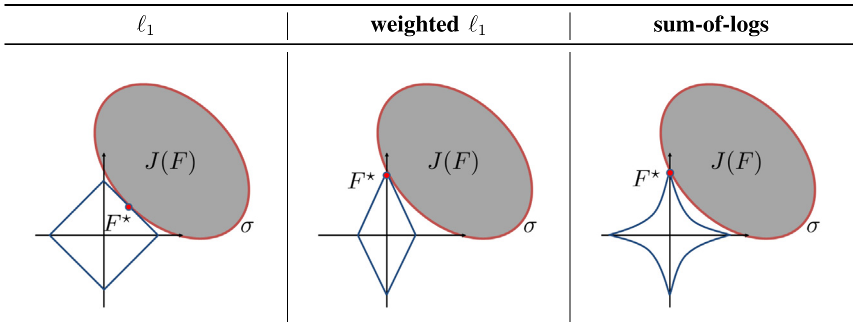

Fig. 7. The solution $F^\star$ of the constrained problem (6) is the intersection of the constraint set $\mathcal{C} := \{F \mid J(F) \le \sigma\}$ and the smallest sub-level set of g that touches $\mathcal{C}$. The penalty function g is the ℓ_1 norm (left); the weighted ℓ_1 norm with appropriate weights (middle); and the nonconvex sum-of-logs function (right).

Approximating **card**(F) with a penalty function $g(F)$ yields

$$\begin{array}{ll} \text{minimize} & g(F) \\ \text{subject to} & J(F) \le \sigma. \end{array} \qquad (6)$$

The solution to (6) is the intersection of the constraint set $\mathcal{C} := \{F \mid J(F) \le \sigma\}$ and the smallest sub-level set of g that touches $\mathcal{C}$; see Fig. 7 for an illustration. In contrast to the ℓ_1 norm whose sub-level sets are determined by the convex ℓ_1 ball, the sub-level sets of the nonconvex sum-of-logs function have a star-like shape.

We note that alternative regularization terms $g(F)$ can be introduced to enforce the communication of only relative information exchange in the distributed controller [116,48] or penalize more nuanced measures of controller complexity [73–76]. Furthermore, in the above framework the elements of the feedback gain matrix were assumed to represent communication links in the distributed controller. Recently, a more general framework where a communication link is a linear function of the elements of the feedback gain matrix was developed in [117]. These more advanced regularization penalties reflect the fact that sparsity should be enforced in a specific set of coordinates.

3.2. Polishing: back to structured optimal design

After having identified the controller architecture, we optimize the closed-loop $\mathcal{H}_2$ performance over the identified structure $\mathcal{S}$. The optimal feedback gain matrix with fixed sparsity patterns $F \in \mathcal{S}$ is obtained by solving the structured $\mathcal{H}_2$ problem (SH2a). The optimality conditions are given by

$$\begin{aligned} (A - B_2 F)^T P + P(A - B_2 F) &= -(Q + F^T R F) \\ (A - B_2 F) X + X(A - B_2 F)^T &= -B_1 B_1^T \\ \left[\left(RF - B_2^T P \right) X \right] \circ I_{\mathcal{S}} &= 0 \end{aligned}$$

where $I_{\mathcal{S}}$ is the *structural identity* (under elementwise matrix multiplication $\circ$) of the subspace $\mathcal{S}$,

$$\{F \circ I_{\mathcal{S}} = F \text{ for } F \in \mathcal{S}\} \Leftrightarrow [I_{\mathcal{S}}]_{ij} = \begin{cases} 1, & F_{ij} \text{ is a free variable} \\ 0, & F_{ij} = 0. \end{cases}$$

The solution is obtained using Newton's method which employs the conjugate gradient scheme that does not require forming or inverting the large Hessian matrix explicitly [68].

4. Generalized problem formulation and classes of convex problems

We next depart from the state-feedback problem and introduce a more general setup where a design variable F specifies a matrix $K(F)$ which modifies the open-loop dynamics,

$$\dot{x} = (A - K(F))\, x + B_1\, d. \qquad (7)$$

Here, K is a linear mapping from a finite-dimensional Hilbert space to the space of $n \times n$ matrices. In the remainder of the paper, we use f and F to differentiate between vector and matricial design variables. This formulation includes the static output-feedback, $K(F) = B_2 F C$, diagonal modifications of open-loop dynamics, $K(f) = \sum f_k D_k$, where f_k are scalars and D_k are given diagonal matrices, and the design of undirected consensus networks, $K(f) = E\, \mathrm{diag}(f)\, E^T$, where f is a vector of edge weights and E is the incidence matrix of the controller graph.

Although this generalized formulation can be cast as a *structured* output-feedback problem, it allows us to draw a more natural connection with controller structure and it is convenient for the development of optimization algorithms. To illustrate these points, we first provide several examples that arise in applications and then summarize classes of problems that admit convex characterizations. These include optimal design of sparse symmetric systems, optimal selection of sensors and actuators, design of diagonal modifications to positive systems, and sparse synthesis of consensus and synchronization networks.

4.1. Applications

4.1.1. Optimal edge addition in consensus networks

Reaching consensus via distributed information exchange across a network has garnered much recent attention [79] with applications ranging from social networks [22,47], to distributed computing networks [19,7], to cooperative control of vehicular formations [53,88,80,104,15,105,67].

A consensus network consists of n nodes which update their states using distributed averaging with their neighbors,

$$\dot{x}_i = \sum_j l_{ij}(x_j - x_i) + d_i$$

where the edge weight l_{ij} is nonzero if node j is connected to node i and d_i is a stochastic disturbance. The aggregate dynamics can be written as,

$$\dot{x} = -L\, x + d$$

where L is a weighted directed graph Laplacian. If the graph is connected and balanced, the unforced network achieves consensus. In the presence of white stochastic disturbances, the node values experience a random walk around the network average.

The problem of adding edges to minimize the steady-state variance amplification of stochastically forced consensus networks is equivalent to minimizing the $\mathcal{H}_2$ norm of the closed-loop system,

$$\begin{aligned} \dot{x} &= -(L + K(f))x + d \\ z &= \begin{bmatrix} Q^{1/2} \\ -R^{1/2}\, K(f) \end{bmatrix} x \end{aligned}$$

where f is a vector of edge weights, and $K(f)$ is the controller graph Laplacian. For undirected networks, $K(f) = E\, \mathrm{diag}(f) E^T$

where E is the incidence matrix of the controller graph. For directed networks, $K(f)=\sum_{i,j} f_{ij} E_{ij}$ where f_{ij} is an edge weight and $E_{ij} \in \mathbb{R}^{n \times n}$ is a given matrix associated with a directed edge from j to i. All of the entries of this matrix are zero apart from the iith and ijth entries which are equal to 1 and -1, respectively. Since the average mode is marginally stable, the state penalty Q must have an eigenvalue at zero with the corresponding eigenvector of all ones, $Q\,\mathbb{1}=0$, and it has to be positive definite on the orthogonal complement of the subspace spanned by $\mathbb{1}$, $Q+(1/n)\,\mathbb{1}\mathbb{1}^T \succ 0$. For example, $Q=I-(1/n)\,\mathbb{1}\mathbb{1}^T$ penalizes mean-square deviation from the network average.

The regularization term can be used to impose additional requirements on the closed-loop network. The added edges may be constrained to be undirected, balanced, or be drawn from a set of feasible edges. Regularization can also be used to promote sparsity in the given set of edge weights or impose additional structural properties.

4.1.2. Optimal design of networks of second order systems

In networks of second order systems,

$$\dot{x}=\left(\begin{bmatrix} 0 & I \\ A_{21} & A_{22} \end{bmatrix} - B_2\, K(F)\, C\right)x+B_1\, d \tag{8}$$

the design of $K(F)$ has several interesting interpretations. When (8) represents a system of masses connected by springs and dampers, $x:=[p^T\; v^T]^T$ is the vector of positions and velocities, whereas A_{21} and A_{22} account for spring and damper connections, respectively. In the system shown in Fig. 2, each mass is connected to its immediate neighbors via a spring and there is no damping. This implies that A_{21} is a tridiagonal matrix and that $A_{22}=0$. This formulation naturally extends to masses connected by networks of springs and dampers and has been recently used for control-oriented modeling of civil engineering structures [108].

System (8) can be also used to model formations of vehicles, networks of coupled LC oscillators, and the linearized dynamics of the swing equation. For the vehicular formations, $A_{21}=0$ and $A_{22}=-\kappa I$, where $\kappa \geq 0$ is the linearized drag coefficient per unit mass. For the LC oscillators, A_{21} is a diagonal matrix and A_{22} is the graph Laplacian of conductive links between the oscillators. For the linearized swing equation, which models the evolution of generator rotor angles in power networks, the matrix A_{21} provides the coupling between generators and A_{22} is a diagonal matrix.

For $B_2=[0\; I]^T$, $C=I$, and $K(F)=F$ we recover the standard state-feedback problem. Similarly, for $C=[I\; 0]$ or $C=[0\; I]$, we obtain the static-output feedback problem in which the design variable $K(F)=F$ only modifies elements of the matrices A_{21} or A_{22}, respectively. The setup in which the designer only has the ability to add spring-like links between the nodes amounts to setting $B_2=[0\; I]^T$, $C=[I\; 0]$, and $K(f)=E\,\mathrm{diag}(f)E^T$ in (8). This situation arises in the design or tuning of spring constants in flexible structures [46,86] and in the introduction of additional relative angle information exchange in power networks [114]. Here, the matrix $K(f)$ represents the added structural coupling or coupling between the generator angles. In contrast, the setup in which the designer can add resistive or damping links between the nodes, e.g., when connecting LC oscillators by resistors [40], can be recovered by setting $B_2=[0\; I]^T$, $C=[0\; I]$, and $K(f)=E\,\mathrm{diag}(f)E^T$ in (8).

4.1.3. Diagonal modifications of linear systems

We next describe two classes of problems in which the system dynamics (7) is modified in a fully decentralized fashion,

$$K(f)=\sum_k f_k\, D_k$$

where D_k are given diagonal matrices and f_k are scalar design variables.

Combination drug therapy design for HIV treatment

The replication, mutation, and drug treatment dynamics of a population of distinct HIV mutants can be modeled as a positive system [51,56],

$$\begin{aligned}\dot{x}&=\left(A-\sum_k f_k\, D_k\right)x+B_1\, d\\ z&=C\,x\end{aligned} \tag{9}$$

where A is a Metzler matrix (all off-diagonal elements are nonnegative), B_1 and C are matrices with nonnegative entries, the matrices D_k are diagonal, and the scalars f_k are the design variables. Positive systems are systems for which the state $x(t)$ is nonnegative for all times when the initial condition and disturbances are nonnegative [41]. Since mutation rates cannot be negative, the combination drug therapy problem obeys these assumptions. Here, the ith component of the state vector x represents the population of the ith HIV mutant, the diagonal elements of A represent the replication rates of the mutants, and each off-diagonal element A_{ij} represents the rate of mutation from mutant j to mutant i. Each element f_k of the design variable f represents the dose of the kth drug and the iith entry of the diagonal matrix D_k specifies how efficiently drug k kills the ith HIV mutant.

Leader selection in consensus networks

Controllability of networks has recently emerged as an important paradigm in network science [93,70,18,90]. In this, it is important to identify the set of nodes through which the network is easily influenced. It has been shown that the so-called leader selection problem can be used as a proxy for identifying important nodes in the network [16]. Furthermore, in some applications, it is possible to augment relative information exchange between the nodes in the consensus network with *absolute* information at certain nodes [91,43,69,16,23]. For example, in vehicular formations where each vehicle measures relative distance from its neighbors via ranging devices, absolute information can be acquired by equipping certain vehicles with GPS devices.

In such a setup, the consensus dynamics are modified to,

$$\dot{x}_i=\sum_j l_{ij}\,(x_j-x_i)-f_i\, x_i+d_i$$

where $f_i>0$ if the ith node is the leader and $f_i=0$ otherwise. By taking $A=-L$ and $D_k=\mathrm{e}_k\mathrm{e}_k^T$ in (9), where L is the graph Laplacian of the network and e_k is the standard kth unit vector in $\mathbb{R}^n$, we recover the leader selection problem.

4.2. Classes of convex problems

The design of F in this generalized formulation can be challenging. Even in the absence of structural constraints and regularizers, the change of variables from F to (X,Y) discussed in Section 2.2 is not always possible because the mapping K is not necessarily invertible. Furthermore, even determining stabilizability for this problem is, in general, NP hard; this follows from the inclusion of static output-feedback as a special case [45].

These challenges motivate the identification of classes of convex problems so that stabilizability can be established, the ‘centralized’ solution can be identified, and globally optimal solutions of the regularized and structured control problems can be efficiently computed.

4.2.1. Optimal design of symmetric systems

For symmetric systems, the $\mathcal{H}_2$ norm can be expressed as a convex function of F [25]. When $B_1=I$, A and $K(F)$ are symmetric, and $A-K(F)$ is Hurwitz, the controllability gramian of system (7)

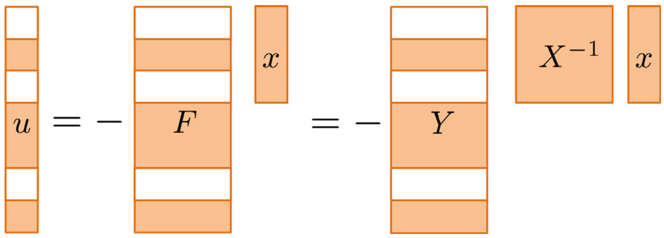

Fig. 8. Equivalence between the row-sparsity of F and the row-sparsity of $Y := FX$.

can be explicitly expressed as,

$$X = -\frac{1}{2}\,(A - K(F))^{-1}.$$

For the regulated output

$$z = \begin{bmatrix} Q^{1/2} \\ -R^{1/2} K(F) \end{bmatrix} x,$$

the $\mathcal{H}_2$ norm is determined by

$$J(F) = \frac{1}{2}\,\text{trace}\Big(\big(Q + K^T(F) R\, K(F)\big)(K(F) - A)^{-1}\Big).$$

Through the use of the Schur complement, the optimal control problem can be cast as an SDP,

$$\begin{array}{ll} \underset{F,Z}{\text{minimize}} & \frac{1}{2}\,\text{trace}\,(Z) \\ \text{subject to} & \begin{bmatrix} Z & \begin{bmatrix} Q^{1/2} \\ -R^{1/2}\,K(F) \end{bmatrix} \\ (\cdot)^T & K(F) - A \end{bmatrix} \succcurlyeq 0 \end{array} \tag{10}$$

and, since F remains as an optimization variable, sparsity-promoting or other convex regularizers can be readily included in the problem formulation. For small problems, the resulting SDP formulation can be solved efficiently using available general-purpose solvers and for larger problems customized algorithms can be developed. Although symmetric systems represent a limited class of problems, stability guarantees and performance bounds for non-symmetric systems have been recently derived using the symmetric components of matrices A and $K(F)$ [25,29].

Optimal design of edges in undirected consensus networks

For the problem of adding edges to an undirected consensus network with graph Laplacian $L = L^T$, we have $A = -L$, $B_1 = I$, and $K(f) := E\,\text{diag}\,(f)\,E^T$. Here, $K(f)$ is the graph Laplacian of the controller, E is the incidence matrix of the controller graph, f is a vector of the added edge weights, and the objective is to minimize variance amplification of the closed-loop network. As in Section 4.2.1, the nullspace of Q is $\mathbb{1}$ and Q is positive definite on $\mathbb{1}^\perp$. Since the matrix $L + K(F)$ has an eigenvalue at 0, the resulting SDP has a slightly different form relative to (10),

$$\begin{array}{ll} \underset{f,Z}{\text{minimize}} & \frac{1}{2}\,\text{trace}\,(Z) + \gamma \sum_i w_i\,|f_i| \\ \text{subject to} & \begin{bmatrix} Z & \begin{bmatrix} Q^{1/2} \\ -R^{1/2}\,E\,\text{diag}\,(f)\,E^T \end{bmatrix} \\ (\cdot)^T & L + E\,\text{diag}\,(f)\,E^T + (1/n)\,\mathbb{1}\mathbb{1}^T \end{bmatrix} \succcurlyeq 0. \end{array} \tag{11}$$

To promote a limited number of added edges, the regularization term is taken to be an ℓ_1 penalty on the edge weights. Note that this formulation allows for the introduction of arbitrary convex regularizers on f. For small problems, standard SDP solvers can be used to compute the optimal solution. For large networks, efficient customized algorithms that exploit the structure of the sparsity-promoting undirected edge addition problem have been recently developed in [48–50].

4.2.2. Optimal actuator and sensor selection

The design of an optimal state-feedback controller which uses a limited number of available actuators can be cast as a convex problem. When the ith row of the feedback gain matrix F is identically equal to zero, the ith control input is not used. Thus, obtaining a control law which uses only a subset of available actuators can be achieved by promoting row-sparsity of F.

Although, as noted in Section 2.2, it is in general difficult to establish the relation between the sparsity structures of F and $Y := FX$, the row-sparsity structure represents a notable exception. This is because the ith row of F is equal to zero if and only if the ith row of Y is equal to zero [92]; see Fig. 8 for an illustration. Thus, by augmenting the $\mathcal{H}_2$ performance metric with a sparsity-promoting penalty on the rows of Y, $\gamma \sum_i w_i\, \| e_i^T Y \|_2$, the problem of optimal actuator selection can be formulated as

$$\begin{array}{ll} \underset{X,Y}{\text{minimize}} & \text{trace}\,(Q\,X) + \text{trace}\,(R\,Y X^{-1}\,Y^T) + \gamma \sum_{i=1}^{m} w_i\, \| e_i^T Y \|_2 \\ \text{subject to} & (A\,X - B_2\,Y) + (A\,X - B_2\,Y)^T + B_1 B_1^T = 0 \\ & X \succ 0 \end{array}$$

and cast as an SDP via the use of Schur complement and the proper treatment of the regularization term. Since general SDP solvers scale poorly with problem dimension, a customized algorithm based on ADMM was developed in [28] to exploit problem structure and efficiently compute the optimal solution. It is also worth noting that the problem of designing a Kalman filter which uses a limited number of available sensors is dual to the problem of optimal actuator selection and it thus admits similar convex characterization [28].

4.2.3. Diagonal modifications of positive systems

Leveraging recent results on diagonal modifications to positive systems [94,17], the convexity of the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms of (9) with respect to the vector f (with elements f_k) was established in [24]. This problem is not SDP representable and its solution is found via a customized algorithm developed in [24]. This formulation does not include a traditional measure of the control effort in the performance output z. It instead limits the control effort via introduction of suitable regularization terms on the design variable f [24].

In many applications, penalizing f directly is physically meaningful. Regularization of f in the drug therapy problem can directly impose clinically relevant constraints including budget of drugs, $\sum f_k \le T$, or maximum drug doses, $f_k \le T_k$. Convex constraints can also be used to promote logical conditions arising from drug–drug interactions, such as mutual exclusivity of drugs j and i via $f_i + f_j \le T$, or drug j requiring drug i via $f_j \le f_i$ [58]. For the leader selection problem, regularization of f can promote identification of a sparse set of leaders or limit the feedback gains on the absolute position that the leaders are using.

4.2.4. Synchronization of oscillators

The problem of coupling oscillators with the same resonance frequency ω via a network of conductances,

$$\dot{x} = \begin{bmatrix} 0 & I \\ -\omega^2 I & -E\,\text{diag}\,(f)\,E^T \end{bmatrix} x + d$$

can be cast as an SDP; see [40] for details. This structure arises in applications that include synchronization of power networks and Kuramoto oscillators [54,31,30].

5. Examples

We next provide several examples to illustrate the utility of our framework. The undirected consensus network is from [49], the sensor selection example is from [28], the wide-area control of a power networks has been studied in [113], and the combination drug therapy example is from [24]. Additional examples, along with MATLAB source codes, are available at www.ece.umn.edu/users/mihailo/software/lqrsp/.

5.1. Disconnected consensus network

An undirected consensus network is generated from $n=50$ randomly distributed nodes in 10×10 unit box. Two nodes are connected with unit edge weight if the Euclidean distance between them is less than 2 units. The example we present here is not connected, and at least two additional undirected edges are required for this network to achieve consensus.

We approach the undirected edge addition problem described in Section 4.2.1 for a controller graph with $m=1094$ potential edges. This is achieved by imposing weighted ℓ_1 regularization on the vector of edge weights for 200 logarithmically spaced values of $\gamma \in [10^{-3}, 2.5]$ using the path-following iterative re-weighted algorithm as a proxy for inducing sparsity [14]. We set the weights to be inversely proportional to the magnitude of the solution f at the previous value of γ and initialize weights for $\gamma = 10^{-3}$ using the optimal centralized vector of the edge weights. Topology identification is followed by the polishing step that computes the optimal edge weights.

Fig. 9 illustrates topologies of the plant (blue lines) and the controller (red lines) graphs for four values of γ. As expected, larger values of γ yield sparser controller graphs. Since the plant graph has three disconnected subgraphs, at least two edges in the controller are needed to make the closed-loop network connected.

Fig. 10 shows that the number of nonzero elements in the vector of the edge weights f decreases and that the closed-loop performance deteriorates as γ increases. In particular, Fig. 10c illustrates the optimal tradeoff curve between the $\mathcal{H}_2$ performance loss (relative to the optimal centralized controller) and the sparsity of the vector f. For $\gamma = 2.5$, only four edges are added. Relative to the optimal centralized vector of the controller edge weights f_c, the identified sparse controller in this case uses only 0.37% of the edges, i.e., $\textbf{card}(f)/\textbf{card}(f_c) = 0.37\%$, and achieves a performance loss of 82.13%, $(J - J_c)/J_c = 82.13\%$. Here, f_c is the solution to the sparsity-promoting optimal control problem with $\gamma = 0$ and the pattern of non-zero elements of f is obtained by solving the problem formulated in Section 4.2.1 with $\gamma = 2.5$ via the path-following iterative re-weighted algorithm.

5.2. Sensor selection for flexible aircraft

One barrier in reducing aircraft weight in order to improve fuel efficiency is that lighter airframes are more flexible and thus susceptible to vibrational instabilities [11]. These instabilities, known as flutter, were behind the famous Tacoma Narrows Bridge collapse and have been identified as the likely cause of the loss of NASA's Helios Prototype aircraft [87].

Recent work has sought to approach this problem by actively damping flutter instabilities [4]. Since active control requires

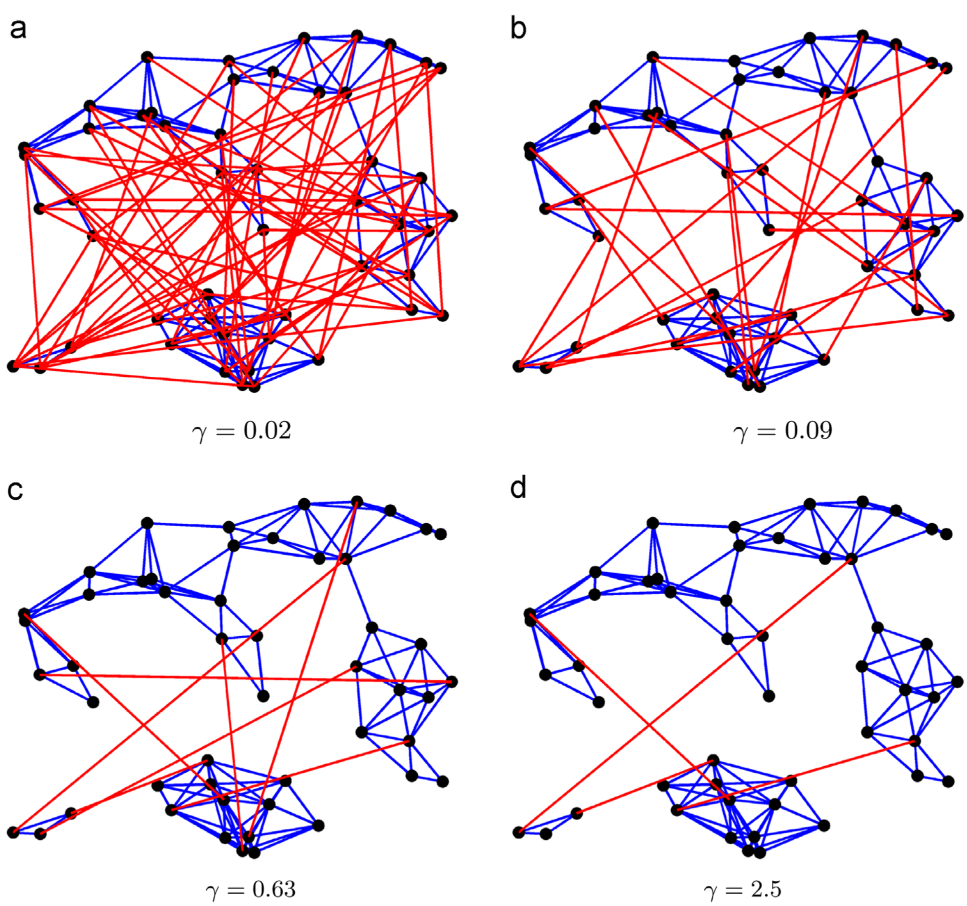

Fig. 9. Topologies of the plant (blue lines) and controller graphs (red lines) for an unweighted random network with three disconnected subgraphs. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

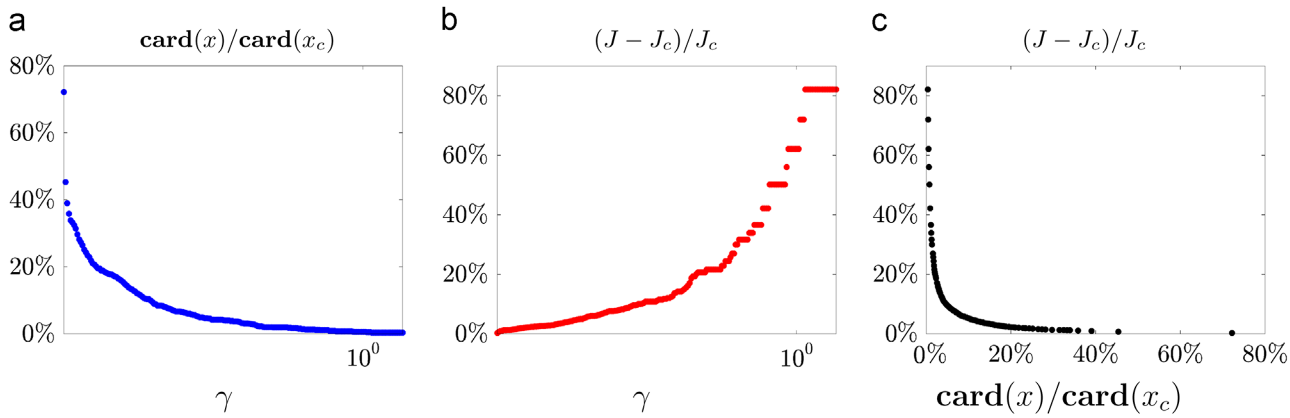

Fig. 10. (a) Sparsity level; (b) performance degradation; and (c) the optimal tradeoff curve between the performance degradation and the sparsity level of optimal sparse f compared to the optimal centralized vector of the edge weights f_c. The results are obtained for unweighted random disconnected plant network with topology shown in Fig. 9.

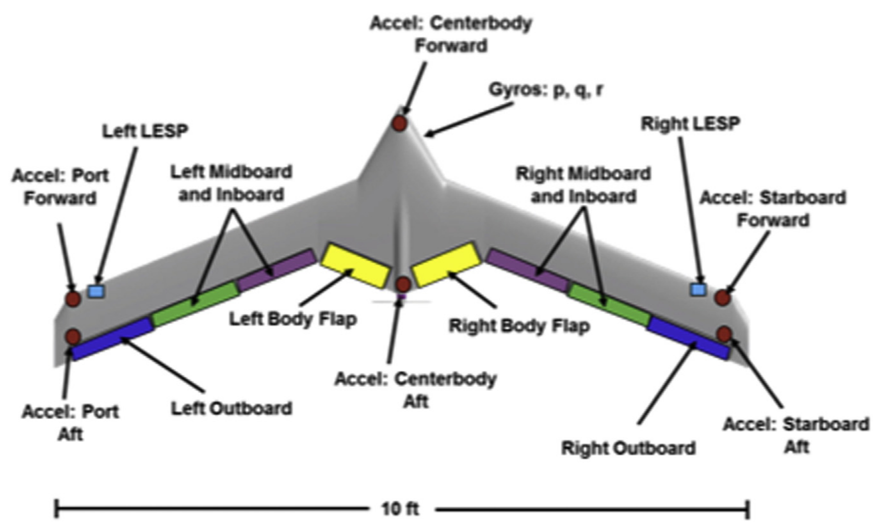

Fig. 11. Body Freedom Flutter flexible wing testbed aircraft.

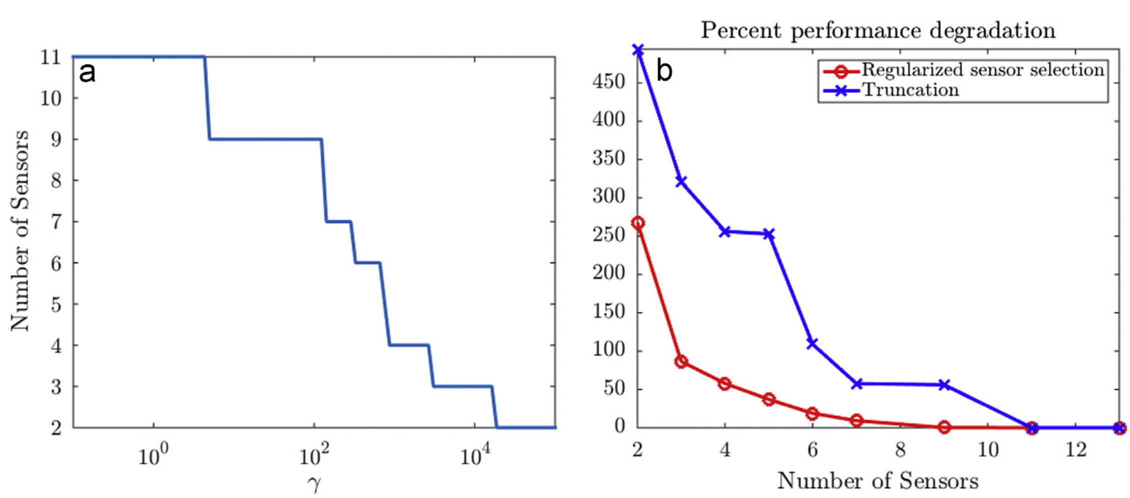

Fig. 12. (a) Number of sensors as a function of the sparsity-promoting parameter γ; and (b) performance comparison of the Kalman filter associated with the sets of sensors resulting from the regularized sensor selection problem and from truncation.

reliable detection of instabilities, selection of sensors is an important challenge. For the Body Freedom Flutter test aircraft shown in Fig. 11 [81], we use the dual formulation to the actuator selection approach described in Section 4.2.2 in conjunction with iterative re-weighting algorithm to select sparse sets of sensors. Fig. 12 shows the number of sensors as a function of the sparsity-promoting parameter γ and the performance of Kalman filter with the limited sets of sensors [28].

We compare the performance of the Kalman filter corresponding to the sensors selected by our approach to the Kalman filter associated with sensors selected by truncation. For the truncation approach, the Kalman gain matrix corresponding to a set of sensors was computed. The sensor corresponding to the row with the lowest ℓ_2 norm was discarded and the Kalman gain was recomputed for the new set of sensors. This process was repeated iteratively from the full set of sensors to a set of two sensors. Clearly, the regularized sensor selection algorithm described in Section 4.2.2 selects better subsets of sensors than the truncation approach.

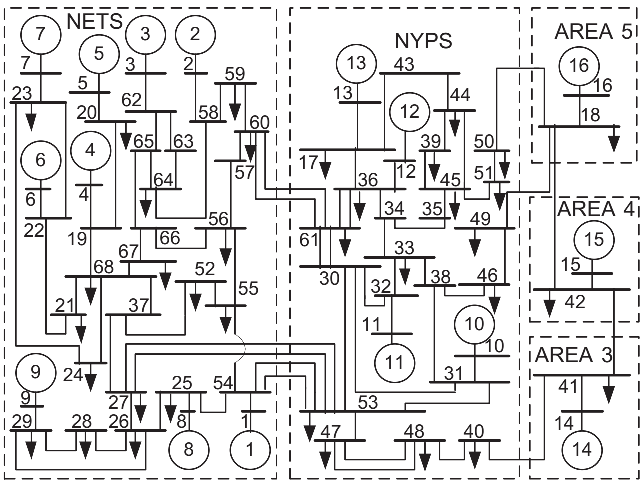

Fig. 13. NETS–NYPS test system.

5.3. Sparsity-promoting wide-area control of a power network

The New England Test System (NETS)–New York Power System (NYPS) example consists of 16 machines, 68 buses, and 5 areas; see Fig. 13. All the generators have fast static excitation system, while generators 1–12 are also equipped with Power System Stabilizers (PSSs); see [89,103] for a detailed description of the model.

The open-loop system is unstable, and PSSs are used for stabilization and to suppress local oscillations. For the wide-area control design, we assume that the PSS inputs are embedded in the open-loop matrix $A \in \mathbb{R}^{147 \times 147}$. In our recent work [113], we have shown that optimal retuning of fully decentralized controllers can guard against local and inter-area oscillations but incorporating limited communication exchange can further improve performance.

Elementwise sparsity is not the appropriate desired structure for this problem. Since the obstacle precluding centralized control is maintaining a communication channel, the marginal cost of sending additional state information between generators that are already connected is low. Moreover, each generator has 'free' access to its own states. This motivates regularization with a *block-sparsity* promoting penalty function which does penalize each generators' access to its own states. Furthermore, since only relative information is

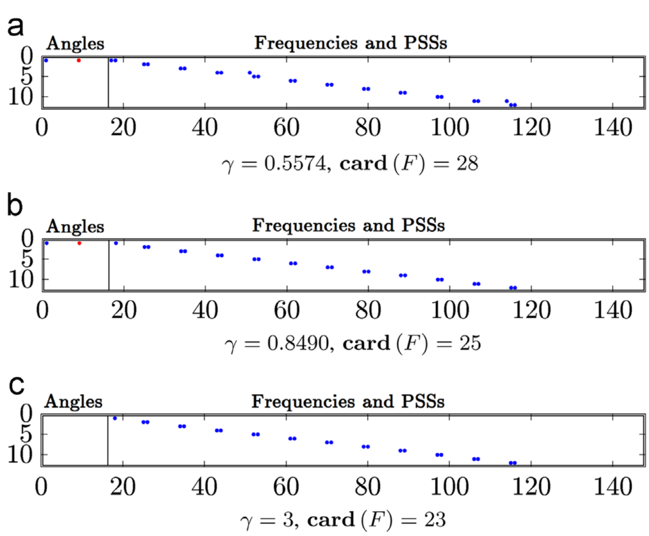

Fig. 14. Sparsity patterns of F resulting from regularizers that promote elementwise sparsity.

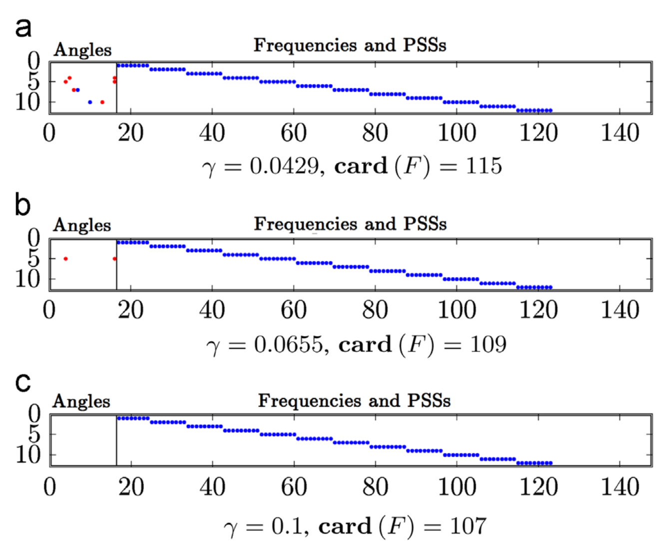

Fig. 16. Sparsity patterns of F resulting from regularizers that promote block sparsity.

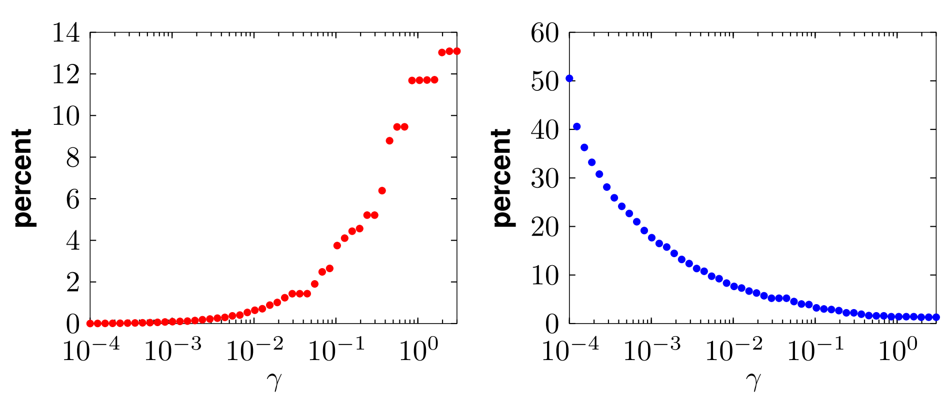

Fig. 15. Performance vs. sparsity ($(J - J_c)/J_c$ vs. $\mathbf{card}(F)/\mathbf{card}(F_c)$) comparison of sparse F and the optimal centralized controller F_c for 50 logarithmically spaced points $\gamma \in [10^{-4}, 3]$.

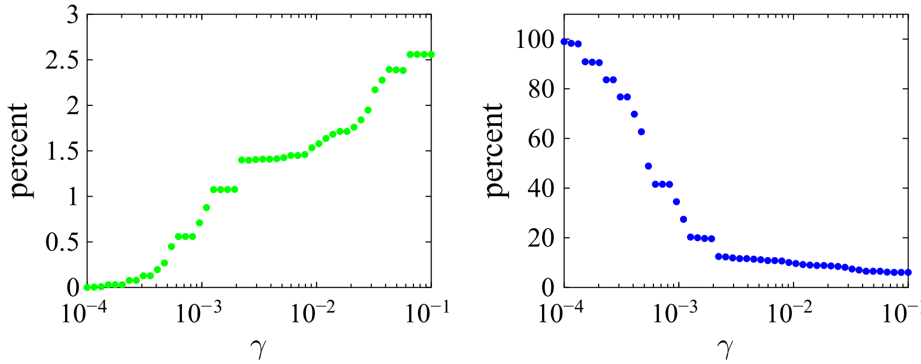

Fig. 17. Performance vs. sparsity comparison ($(J-J_c)/J_c$ vs. **card**(F)/**card**(F_c)) of block-sparse F and the optimal centralized controller F_c for 50 logarithmically spaced points $\gamma \in [10^{-4}, 0.1]$.

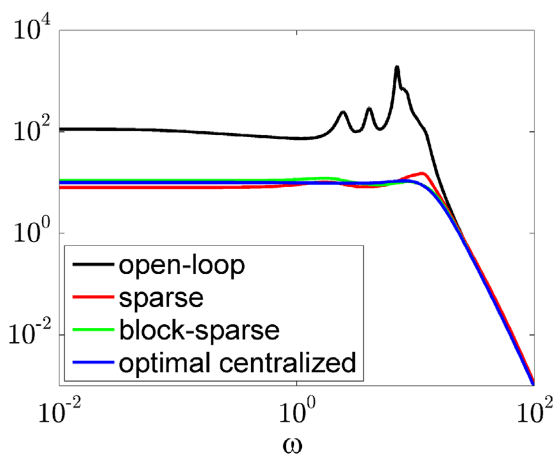

Fig. 18. Power spectral density comparison.

available about rotor angles, the block of the feedback gain matrix mapping θ to u must have a Laplacian structure.

We next provide a brief summary of our computational results.

5.3.1. *Elementwise sparsity*

We first consider an optimal controller whose structure is identified using regularizers that promote elementwise sparsity. Sparsity patterns of the feedback matrix $F \in \mathbb{R}^{12\times147}$ for different values of γ are illustrated in Fig. 14. The blue dots denote information coming from the generators on which the particular controller acts, and the red dots identify information that needs to be communicated from other generators. For $\gamma = 0.5574$ and $\gamma = 0.8490$, the identified wide-area control architecture indicates that the controller of generator 1 needs to have access to the difference between its angle and the angle of generator 9.

When γ is increased to 3, we obtain a fully decentralized controller. Compared to the optimal centralized controller, our fully decentralized controller degrades the closed-loop performance by about 13.09%; see Fig. 15. This fully decentralized controller can be embedded into the local generator excitation system by directly feeding the local measurements to the automatic voltage regulator, thereby effectively retuning the PSS controller.

5.3.2. *Block sparsity*

Three identified block-sparsity patterns of the feedback matrix are shown in Fig. 16. When $\gamma = 0.1$, we obtain a fully decentralized controller structure. The group sparsity-promoting penalty function yields block-diagonal feedback gains that act on the remaining states of generators 1–12. Since no information exchange with generators 13–16 is required, this part of F is implemented in a fully decentralized fashion.

Compared to the optimal centralized controller, a fully decentralized controller with structure shown in Fig. 16c compromises performance by only 2.56% for system of such large dimension; see Fig. 17. We recall that the fully decentralized controller with structure shown in Fig. 14c degrades performance by 13.09%; cf. Fig. 15. Since the block-sparse controller has more degrees of freedom than the elementwise sparse controller, performance improvement does not come as a surprise. We finally note that the jumps in the number of non-zero elements in Fig. 17 are caused by elimination of the entire off-diagonal rows of the feedback gain that acts on the states that exclude generator angles.

5.3.3. *Comparison of open- and closed-loop systems*

We next compare performance of the open-loop system and the closed-loop systems with optimal centralized and fully decentralized sparse and block-sparse controllers. The structures of these fully decentralized controllers are shown in Figs. 14c and 16c, respectively.

Fig. 18 provides a comparison between the power spectral densities of four cases. All three controllers successfully suppress resonant peaks associated with the poorly damped modes and significantly improve performance. We also note that the fully decentralized block-sparse controllers perform almost as well as the optimal centralized controller for high frequencies; for low frequencies, we observe minor performance degradation.

5.4. *Combination drug therapy for HIV treatment*

Consider the HIV combination drug therapy problem described in Section 4.1.3. Following [59,57,55], we study a system with 35 mutants x and 5 drugs f. The sparsity pattern of the A matrix, shown in Fig. 19, corresponds to the mutation pattern and replication rates of the HIV mutants and $K(f)$ specifies the effect of drug therapy.

5.4.1. *Budget constraint*

We first impose a unit budget constraint on the drug doses and solve the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ optimal control problems,

$$\begin{aligned} &\text{minimize} \quad J(f) \\ &\text{subject to} \quad \sum_i f_i = 1, \quad f_i \geq 0, \end{aligned}$$

using proximal gradient and proximal subgradient methods [5,6]. A budget constraint naturally promotes sparsity because of its duality to the ℓ_1 norm. Table 1 contains the optimal doses and illustrates the tradeoff between $\mathcal{H}_2$ and $\mathcal{H}_\infty$ performance.

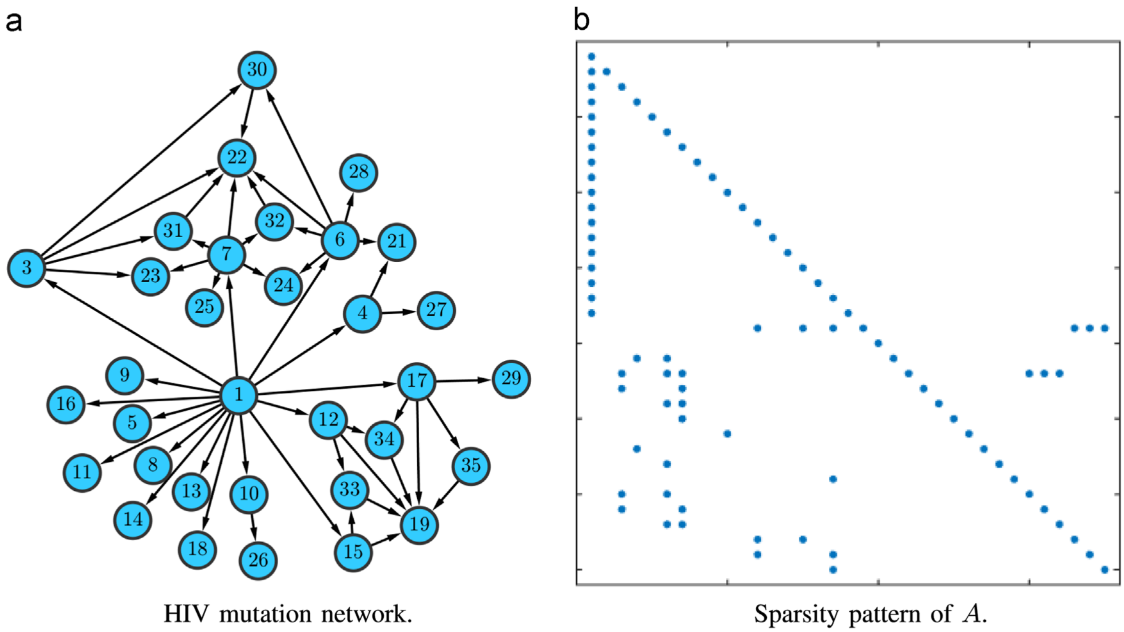

Fig. 19. Mutation pattern in the HIV model.

Table 1
Optimal budgeted doses and the corresponding $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms.

Antibody	$f_{\mathcal{H}_2}$	$f_{\mathcal{H}_\infty}$
3BC176	0.5952	0.9875
PG16	0	0
45-46G54W	0.2484	0.0125
PGT128	0.1564	0
10-1074	0	0
Performance	$f_{\mathcal{H}_2}$	$f_{\mathcal{H}_\infty}$
J_2	0.6017	1.1947
J_∞	0.1857	0.1084

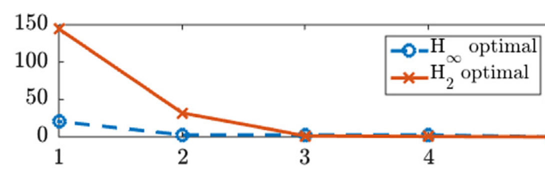

Fig. 20. $\mathcal{H}_2$ and $\mathcal{H}_\infty$ performance degradation (in percents) as a function of the number of drugs.

5.4.2. *Sparsity-promoting framework*

In Algorithm 1, we introduce a quadratic regularization term to limit drug doses and the ℓ_1 regularization term to select sparser sets of drugs. We use a re-weighted ℓ_1 penalty function [14] to select a few drugs and then perform a polishing step to design the optimal doses of selected drugs. In the algorithm, **card** denotes the number of nonzero elements and **sp** denotes the sparsity pattern of the vector f.

Fig. 20 shows performance degradation (in percents) relative to the optimal dose that uses all 5 drugs with $B=C=I$, $R=I$, and γ varying from 0.01 to 10 in 50 logarithmically spaced increments.

Algorithm 1. Sparsity-promoting algorithm for N drugs.

Set $\gamma=0$, $R\succ 0$, $w_i=\mathbb{1}$, $\varepsilon>0$
Structure Identification
While **card** $(f^\gamma)>N$
$f^\gamma=\text{argmin}_f \quad J(f)+f^TRf+\gamma\sum_i w_i\,|f_i|$
$w_i=1/(f_i^\gamma+\varepsilon)$, increase γ
End While
Polishing
$f^\star \;=\; \text{argmin}_f \quad J(f)+f^TRf$
subject to **sp** $(f)\subseteq$ **sp** (f^γ).

6. Concluding remarks

This review article is about the design of controller structures that achieve a balance between performance and some measure of structural complexity. In large-scale dynamical networks, one notion of structural complexity is given by the level of information exchange in the closed-loop system. To obtain structured architectures, we solve regularized versions of the standard $\mathcal{H}_2$ optimal control problem. Such regularization yields a parameterized family of controller architectures that establish a tradeoff between closed-loop performance and the structural properties of the controller. Finally, we design an optimal controller by minimizing the $\mathcal{H}_2$ norm over the identified structure. For large-scale networks of dynamical systems where limited information exchange is desired, our approach yields a parameterized family of communication topologies that gradually transition from all-to-all to decentralized architectures.

Since these problems are nonconvex in general, we have developed algorithms based on the augmented Lagrangian method to obtain locally optimal solutions. Furthermore, we have identified classes of convex problems that arise in the design of symmetric systems, undirected consensus and synchronization networks, optimal selection of sensors and actuators, and decentralized control of positive systems. Finally, we have used examples to illustrate the utility of our approach. Beyond what was reported in the paper, the method has already been applied on a host of problems including technologically relevant wide-area control of power networks [32,33,114], control of high density arrays of micro-cantilevers [77,78], and optimal distributed control for earthquake mitigation in civil engineering structures [108].

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